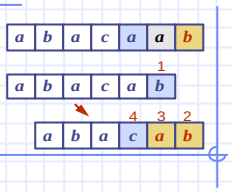
Pattern Matching



Outline and Reading

- Strings (§11.1)
- Pattern matching algorithms (§11.2)
  - Brute-force algorithm
  - Boyer-Moore algorithm
  - Knuth-Morris-Pratt algorithm

Strings

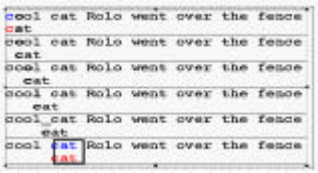
- A string is a sequence of characters
- Examples of strings:
  - Java program
  - HTML document
  - DNA sequence
  - Digitized image
- An alphabet S is the set of possible characters for a family of strings
- Example of alphabets:
  - ASCII
  - Unicode
  - {0, 1}
  - {A, C, G, T}
- Let P be a string of size m
  - A substring $P[i \,..\, j]$ of P is the subsequence of P consisting of the characters with ranks between i and j
  - A prefix of P is a substring of the type $P[0 \,..\, i]$
  - A suffix of P is a substring of the type $P[i \,..\, m-1]$
- Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- Applications:
  - Text editors
  - Search engines
  - Biological research

Brute-Force Algorithm

- The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T, until either
  - a match is found, or
  - all placements of the pattern have been tried
- Brute-force pattern matching runs in time $O(nm)$
- Example of worst case:
  - $T = aaa \ldots ah$
  - $P = aaah$
  - may occur in images and DNA sequences
  - unlikely in English text

Algorithm *BruteForceMatch(T, P)*
Input text T of size n and pattern P of size m
Output starting index of a substring of T equal to P or -1 if no such substring exists
for $i \leftarrow 0$ **to** $n - m$
{ test shift i of the pattern }
$j \leftarrow 0$
while $j < m \wedge T[i + j] = P[j]$
$j \leftarrow j + 1$
if $j = m$
return i { match at i }
else
return -1 { no match }

Brute Force



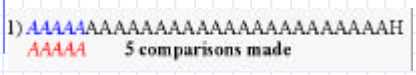
Brute Force-Complexity

- Given a pattern M characters in length, and a text N characters in length...
- Worst case: compares pattern to each substring of text of length M. For example, M=5.
- This kind of case can occur for image data.

Total number of comparisons: M (N-M+1)
Worst case time complexity: O(MN)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- Best case if pattern found: Finds pattern in first M positions of text. For example, M=5.



Total number of comparisons: M
Best case time complexity: O(M)

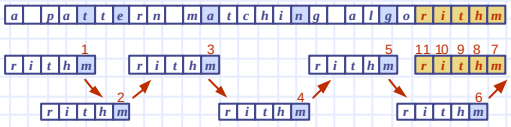
Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- Best case if pattern not found: Always mismatch on first character. For example, M=5.

Total number of comparisons: N
Best case time complexity: O(N)

Boyer-Moore's Algorithm (1)

- The Boyer-Moore's pattern matching algorithm is based on two heuristics
  Looking-glass heuristic: Compare P with a subsequence of T moving backwards
  Character-jump heuristic: When a mismatch occurs at $T[i] = c$
  - If P contains c, shift P to align the last occurrence of c in P with $T[i]$
  - Else, shift P to align $P[0]$ with $T[i+1]$
- Example



Last-Occurrence Function

- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet S to build the last-occurrence function L mapping S to integers, where $L(c)$ is defined as
  - the largest index i such that $P[i] = c$ or
  - -1 if no such index exists
- Example:
  - $S = \{a, b, c, d\}$
  - $P = abacab$

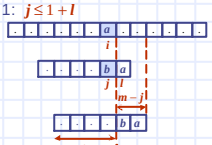
c	a	b	c	d
$L(c)$	4	5	3	-1

- The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- The last-occurrence function can be computed in time $O(m+s)$, where m is the size of P and s is the size of S

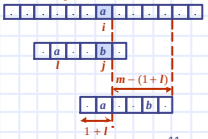
Boyer-Moore's Algorithm (2)

Algorithm *BoyerMooreMatch*(*T*, *P*, *S*)
  $L \leftarrow$ *lastOccurenceFunction*(*P*, *S*)
  $i \leftarrow m - 1$
  $j \leftarrow m - 1$
  repeat
    if $T[i] = P[j]$
      if $j = 0$
        return i { match at i }
      else
        $i \leftarrow i - 1$
        $j \leftarrow j - 1$
    else
      { character-jump }
      $l \leftarrow L[T[i]]$
      $i \leftarrow i + m - \min(j, 1 + l)$
      $j \leftarrow m - 1$
  until $i > n - 1$
  return -1 { no match }

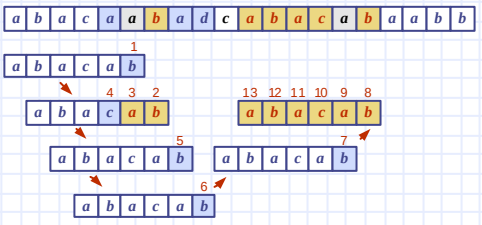
Case 1: $j \leq 1 + l$



Case 2: $1 + l \leq j$

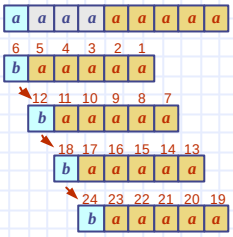


Example



Analysis

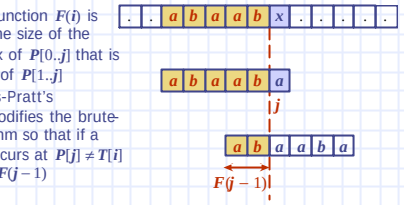
- Boyer-Moore's algorithm runs in time $O(nm + s)$
- Example of worst case:
  - $T = aaa \ldots a$
  - $P = baaa$
- The worst case may occur in images and DNA sequences but is unlikely in English text
- Boyer-Moore's algorithm is significantly faster than the brute-force algorithm on English text



KMP's Algorithm (1)

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The failure function $F(i)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j-1)$

j	0	1	2	3	4	5
$P[j]$	a	b	a	a	b	a
$F(j)$	0	0	1	1	2	3



KMP's Algorithm (2)

- The failure function can be represented by an array and can be computed in $O(m)$ time

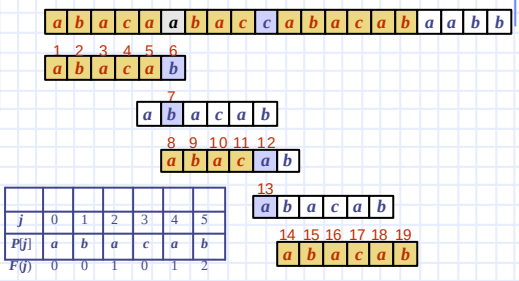
```
Algorithm FailureFunction( P)
    i ← 1
    j ← 0
    F[0] ← 0
    while i < m
        if P[i] = P[j]
            F[i] ← j + 1
            i ← i + 1
            j ← j + 1
        else if j > 0
            j ← F[j − 1]
        else
            F[i] ← 0
            i ← i + 1
    return F
```


KMP's Algorithm (3)

- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2n$ iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time $O(m + n)$

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j − 1]
            else
                i ← i + 1
    return −1 { no match }
```


Example



j	0	1	2	3	4	5
$P[j]$	a	b	a	c	a	b
$F(j)$	0	0	1	0	1	2