WACC is not an expected return of the levered firm

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Version from October 15, 2002

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The model of Modigliani and Miller is one of the cornerstones of modern finance. In their model a tax system generates advantages from debt financing that can be valued using the weighted average cost of capital. Although widely used the concept of cost of capital is usually loosely defined: we provide a simple binomial model showing the counterintuitive result that these cost of capital cannot be interpreted as expected returns of the levered firm.

keywords: cost of capital, capital budgeting

JEL–class.: G 31, D46

One of the cornerstones of modern finance is the Modigliani–Miller theorem (see Modigliani and Miller (1958) and Modigliani and Miller (1963)) on capital structure of the firm. Modigliani and Miller develop a model of a firm having a perpetual cash flow and constant level of debt and show how it can be evaluated using the now well–known WACC formula.

Our focus will be on the question how these weighted cost of capital can be interpreted. Consider a levered firm in time t having value $\widetilde{V}_t^l$. Let $\widetilde{CF}_t$ be the cash flow of the unlevered firm, i.e. except the advantage from debt financing. The textbook discounting rule says that

$$V_0^l = \sum_{t=1}^{\infty} \frac{\mathrm{E}[\widetilde{CF}_t]}{(1+\mathrm{WACC})^t} \,. \tag{1}$$

where WACC are the *cost of capital*. It also seems intuitively clear that this principle can easily be generalized to future points in time to a simple calculation principle for future values of the levered company

$$\widetilde{V}_1^l = \sum_{t=2}^{\infty} \frac{\mathrm{E}[\widetilde{CF}_t|\mathcal{F}_1]}{(1+\mathrm{WACC})^{t-1}} \,. \tag{2}$$

In general, two possible interpretations for these weighted average cost of capital WACC exist

1. Cost of capital in (1) and (2) can be interpreted as (conditional) expected returns. We find a lot of references in the literature stating this point of view.[1]

2. Cost of capital can be interpreted as internal rate of return, i.e. as (any) real number satisfying the equations (1) and (2).[2]

Although the first interpretation seems very intuitiv (nevertheless Modigliani and Miller did not follow this interpretation) we will provide a simple model where our WACC fails to be an expected return of the levered firm. This causes problems if the Modigliani–Miller formula is applied to the capital market line of the CAPM: in this case cost of capital are evidently expected returns.

This directly points to the second interpretation, but things are not that simple. Suppose WACC being any number satisfying only equation (1), what will happen to the valuation equation at $t = 1$? It turns out that in our model (trivially) still a number WACC_1 exists that satisfies the valuation equation (2), but this number is now different from WACC in (1) and does furthermore depend on the state at $t = 1$. This shows that although at time $t = 0$ the weighted average of cost of capital are indeed a device for calculating the value of the levered firm this does not give us a calculation principle that can be used in future points in time: so far WACC is a discount factor restricted to the one particular valuation equation (1) and cannot be used in (2) or any later point in time. Our results do not falsify the theory of Modigliani–Miller but show the need for carefully examining the underlying assumptions of a discounted cashflow model.

I. The Model

Let r_f be the riskless interest rate. We consider a binomial model, i.e. a sequence of iid random variables Y_t having only two realizations: $Y_t(\omega) \in [1+u, 1-u]$. Our firm has a cash flow that increases or decreases by Y_t,

$$\widetilde{C}F_t = \widetilde{C}F_{t-1} \cdot Y_t$$

starting with $CF_0 = 1$. The value of the company is given by the expression

$$\widetilde{V}_t^u = \frac{1-u^2}{r_f + u^2}\,\widetilde{C}F_t. \tag{3}$$

We first have to show that our model is free of arbitrage. To this end it is sufficient to show that there is an equivalent martingale measure such that the expected rate of return of the company is equal to the riskless interest rate. We use

$$Q\Big(Y_t(\omega) = 1+u\Big) = \frac{1-u}{2}, \quad Q\Big(Y_t(\omega) = 1-u\Big) = \frac{1+u}{2}\,.$$

Then the return from holding the share for one period is under the probability Q

$$\begin{aligned}
&\mathrm{E}_Q[\widetilde{V}_t^u + \widetilde{C}F_t|\mathcal{F}_{t-1}] = \\
&\qquad = \left(\frac{1-u^2}{r_f+u^2}+1\right)\widetilde{C}F_{t-1}\left(\frac{1-u}{2}(1+u) + \frac{1+u}{2}(1-u)\right) \\
&\qquad = \frac{1+r_f}{r_f+u^2}\,(1-u^2)\,\widetilde{C}F_{t-1} \\
&\qquad = (1+r_f)\widetilde{V}_{t-1}^u
\end{aligned}$$

verifying that our model is indeed free of arbitrage.

Let the subjective probability measure of the investor be

$$P\Big(Y_t(\omega) = 1+u\Big) = \frac{1}{2}, \quad P\Big(Y_t(\omega) = 1-u\Big) = \frac{1}{2}$$

We now turn to the model of Modigliani and Miller. The levered firm has cash flows before taxes and interest identical to $\widetilde{C}F_t$. There is a firm income tax τ, interest reduces tax. The levered firm maintains a constant amount of debt D. Since the tax advantages from debt are certain we have at time t

$$\widetilde{V}_t^l = \widetilde{V}_t^u + \sum_{s=t+1}^{\infty}\frac{\tau r_f D}{(1+r_f)^{s-t}} = \widetilde{V}_t^u + \tau D. \tag{4}$$

This valuation formula holds regardless whether cost of capital are interpreted as expected returns or as internal rate of return.

A. Cost of Capital as Expected Returns

Let us assume that cost of capital are expected returns given the subjective probability measure P. Then for the unlevered firm

$$\begin{aligned} k^u &:= \frac{\mathrm{E}[\widetilde{V}^u_{t+1} + \widetilde{C}F_{t+1}|\mathcal{F}_t]}{\widetilde{V}^u_t} - 1 \\ &= \frac{\frac{1+u+1-u}{2}(1 + \frac{r_f+u^2}{1-u^2})\widetilde{V}^u_t}{\widetilde{V}^u_t} - 1 \\ &= \frac{1+r_f}{1-u^2} - 1. \end{aligned}$$

The cost of equity of the unlevered firm are constant as assumed in the Modigliani–Miller world. From the definition of cost of capital we get that the value of the unlevered firm can also be written as

$$\widetilde{V}^u_t = \sum_{s=t+1}^{\infty} \frac{\mathrm{E}[\widetilde{C}F_s|\mathcal{F}_t]}{(1+k^u)^{s-t}} = \frac{\widetilde{C}F_t}{k^u}. \tag{5}$$

We now turn to the weighted average cost of capital. Since these cost of capital are also expected returns

$$\begin{aligned} \mathrm{WACC} &:= \frac{\mathrm{E}[\widetilde{V}^l_t + \widetilde{C}F_t|\mathcal{F}_{t-1}]}{\widetilde{V}^l_{t-1}} - 1 \\ &= \frac{\mathrm{E}[\widetilde{V}^u_t + \tau D + \widetilde{C}F_t|\mathcal{F}_{t-1}]}{\widetilde{V}^u_{t-1} + \tau D} - 1 \\ &= \frac{(1+k^u)\widetilde{V}^u_{t-1} + \tau D}{\widetilde{V}^u_{t-1} + \tau D} - 1 \\ &= \frac{k^u}{1 + \tau\frac{D}{\widetilde{V}^u_{t-1}}}. \end{aligned}$$

If WACC and k^u are constant, the last equation is a contradiction. The left hand side is not a random variable although the right hand side depends on time and states of nature. In our example the assumptions of Modigliani and Miller are contradictious: if debt is constant the WACC cannot be the same at every period in time.

To understand the underlying difficulty let us return to the last equation. Assume the value of the firm would not be a random variable and furthermore constant. Then

$$\begin{aligned} \text{WACC} &= \frac{k^u}{1 + \tau \frac{D}{V^l - \tau D}} \\ &= \frac{k^u}{1 + \frac{\tau l}{1 - \tau l}} = k^u (1 - \tau l) \end{aligned}$$

and this is the classical Modigliani–Miller formula. Our problem stems from the fact that the value of the levered firm is uncertain. In this case a constant WACC implies that the future leverage ratios are deterministic – and hence debt cannot be deterministic too.

B. Cost of Capital as Internal Rate of Return

We already mentioned that cost of capital could also be interpreted as internal rate of return. Let the weighted average cost of capital are defined as a real number that satisfies (1). Let us turn to $t = 1$. WACC_1 is now given as the number satisfying (2). This cost of capital has to satisfy

$$\widetilde{V}_1^l = \sum_{t=2}^{\infty} \frac{\text{E}[\widetilde{CF}_t | \mathcal{F}_1]}{(1 + \text{WACC}_1)^{t-1}} = \frac{\widetilde{CF}_1}{\text{WACC}_1}.$$

Since we know the value of the levered firm by (4) and since the conditional expectation is constant this implies

$$\frac{\widetilde{CF}_1}{k^u} + \tau D = \frac{\widetilde{CF}_1}{\text{WACC}_1}$$

or

$$\mathrm{WACC}_1 = \frac{k^u}{1 + \tau k^u \frac{D}{\widetilde{CF}_1}}.$$

If we want to use a constant weighted average cost of capital (in $t = 1$) for discounting the expected cash–flows of the firm, this WACC obviously does depend on time and state of nature in $t = 1$. In particular it will not be the same WACC we used for the calculation of V_0^l at time $t = 0$. The same holds for any other point in time $t > 1$.

II. Conclusion

A simple example showed that the celebrated weighted average cost of capital of the Modigliani–Miller world cannot be interpretated as expected returns. A second possible interpretation of WACC as internal rate of returns also caused problems: these WACC will be state–dependent and time–dependent as well. Hence, they cannot be used as a simple valuation device as any textbook formula suggests.

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Notes

[0]The author thanks the *Verein zur Förderung der Zusammenarbeit von Lehre und Praxis am Finanzplatz Hannover e.V.* for financial support and Jörg Laitenberger for many helpful comments.

[1]Miles and Ezzell (1980) show in their section 3 that WACC is the expected rate of return on the firm. Copeland and Weston (1990, p. 401) use in their capital budgeting formula the term “rate of return” instead of cost of capital. In Brealey and Myers (1996, p. 27) “the discount rate is determined by rates of return prevailing in capital markets”. de Matos (2001, p.43) defines cost of capital explicitly as expected returns.

[2]See for example Brealey and Myers (1996, p. 532).