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STATISTIC OF QUASI-PERIODIC SIGNAL WITH RANDOM PERIOD - FIRST APPLICATION ON VOCAL CORDS OSCILLATION

KROUPA Lukáš (CZ), VÁVRA František (CZ), NOVÝ Pavel (CZ)

Abstract. This paper will introduce problem of statistics of quasi-periodic signal in relation to detection of the vocal cords pathology from audio recording. Distribution function of period lengths and its relation to distribution function of immediate frequencies is defined and application on vocal cord diagnostic by classification of periods and frequencies to common (normal) and anomalous is devised.

Keywords: vocal cords, quasi-periodic signal, early diagnosis, random period, statistics, probability model

Mathematics subject classification: Primary 62P10; Secondary 62F10, 62F25

1 Introduction

Quasi-periodic signal is assumed to be continuous bounded signal which repeatedly crosses given level (e.g. zero level) in direction from below to above this level. It's apparent this definition is loose and heuristic, but obvious when applied. See figure 1.

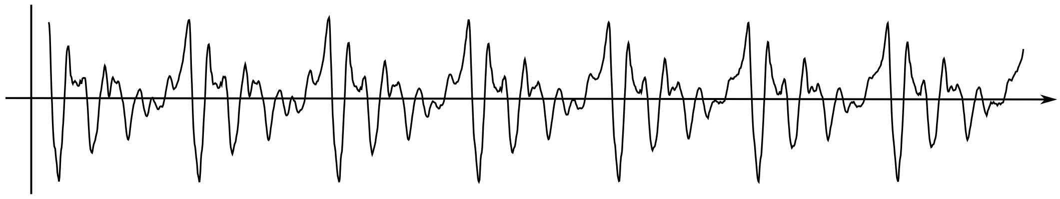

Fig. 1. Example of quasi-periodic signal

The problem of random periods is mainly studied in field of signal processing (for example see [2]). For our problem of vocal cords diagnostics from audio recording[1] we can define

[1]For problem definition and details see [1]

three sub-problems. First, description and classification of length of individual periods and representation in frequency domain. Second, obtaining of amplitude (shape) parameters of individual oscillations. Third problem is "measurement" of deviation of real oscillation from it's estimated ideal shape. The first sub-problem is subject of this paper. We will assume, we can measure period of oscillation[2]. We have random selection of observed oscillation lengths (periods) with given scope, that is the distance between consecutive transition from below to above the zero level. We will focus on signals, where one "period" is composed of sum of components[3], therefore the assumption of asymptotic normal probability distribution is reasonable (see [3], page 376).

2 Distribution function of frequency and it's relation to distribution function of period lengths

Distribution function and density function of period length $T > 0[sec]$ will be denoted as $G_t(x)$, $g_t(x)$ and assume the values are known. Than distribution function of frequency $f = \frac{1}{T} > 0[1/sec]$ will be

$$G_f(x) = P(f \leq x) = P(\frac{1}{T} \leq x) = P(T \geq \frac{1}{x}) = 1 - P(T \leq \frac{1}{x}) = 1 - G_t(\frac{1}{x})$$

, in summary $G_f(x) = 1 - G_t(\frac{1}{x})$ from which the density is

$$g_f(x) = \frac{1}{x^2} g_T(\frac{1}{X}) \tag{1}$$

For the first two moments of frequency f probability description, following would be valid if these moments exist[4]:

$$E(f) = \int_0^{+\infty} \frac{x}{x^2} g_T(\frac{1}{x})dx = \int_0^{+\infty} \frac{1}{x} g_T(\frac{1}{x})dx = \int_0^{+\infty} \frac{1}{y} g_T(y)dy = E_T(\frac{1}{T})$$

summarized as $E(f) = E(\frac{1}{T})$,

$$E(f^2) = \int_0^{+\infty} \frac{x^2}{x^2} g_T(\frac{1}{x})dx = \int_0^{+\infty} g_T(\frac{1}{x})dx = \int_0^{+\infty} \frac{1}{y^2} g_T(y)dy = E_T(\frac{1}{T^2})$$

and accordingly $\sigma^2(f) = \sigma_T^2(\frac{1}{T})$. To avoid problems caused by existence or non-existence of moments, quantile replacement of these moments is used.

For this reason and because of the normality we will assume continuous distribution function $f(x)$ for $x > 0$. We can infer that $G_T(x)$ for $x > 0$ is continuous as well. Onward we will denote $z_p \stackrel{def}{=} G_z^{-1}(p)$ for $0 < p < 1$ and z_p is p quantile of random variable z, i.e. $z_p = inf\{x \in R : p \leq G_z(x)\}$.

[2]For description of signal pre-processing and period length detection see [1], page 23

[3]That is the case for acoustic signal produced by vocal cords.

[4]Transformation of period distribution to frequency distribution might lead to distribution without existence of some of the moments. Even for uniform distribution of periods on interval $(0, T_{max})$ or often used exponential distribution of period duration. Non-existence of moments might not be just academical issue. Fundamentals of this problems are in relation $f = \frac{1}{T}$

3 Quantiles and relation of $G_T(T_p) = p$ and $G_f(f_p) = p$

Let $0 < p < 1$ and $G_T(T_p) = p, G_f(f_p) = p$ from which

$$p = 1 - G_T(\frac{1}{f_p}) \Rightarrow \frac{1}{f_p} = G_T^{-1}(1-p) = T_{1-p}$$

, summarized as

$$f_p = \frac{1}{T_{p-1}} \tag{2}$$

By that we have obtained relation between quantiles of period length and immediate frequency. It might be difficult to get (estimate) one of both f_p, T_{1-p} quatiles in some cases (statistical, estimative). Hence this simple inference, which connects quatiles f_p and T_{1-p}. If following applies

$$G_T(T_p) = G_f(f_p) = 1 - G_T(\frac{1}{f_p} \Rightarrow G_T(\frac{1}{f_p}) = 1 - G_t(T_p) \Rightarrow \frac{1}{f_p = G_T^{-1}(1 - G_T(T_p))}$$

, again summarized:

$$f_p = \frac{1}{G_T^{-1}(1 - G_T(T_p))} \tag{3}$$

then medians $T_{0,5}$, $f_{0,5}$ can be used as replacement values. To describe variability the quartile ranges $T_{0,75} - T_{0,25}$, $f_{0,75} - f_{0,25}$ can be used, or alternatively the width of zone equivalent to σ zone above the median for normal distribution (for technical comparisons), i.e. $T_{0,84134} - T_{0,5}$ and $f_{0,84134} - f_{0,5}$. Obviously this form of variability comparison might be problematic for unsymmetrical distributions. Another alternative might be width of $\pm 3\sigma$ zone in symmetrical position of normal distribution $\frac{T_{0,99865} - T_{0,00135}}{6}$, $\frac{f_{0,99865} - f_{0,00135}}{6}$. These values are equal to value of σ in case of normal distribution. Consequently statistical inference is now possible.

4 Application on vocal cords oscillation

Fundamental prerequisite for vocal cord diagnostics based on audio signal is classification of oscillations (in this case length of their periods and immediate frequencies) to common (standard) and anomalous. We assume phonation recording of vowel "a" ("ááá…"). This process can be divided in several steps:

1. Using statistical methods create probability description of period length.
   a) Elimination of identical values by addition of noise. (We are working with model of continuous random variables.)
   b) Normality test of period lengths set. See appendix 7 for one of possible methods.
   c) Exclusion of observations from this set, which do not conform to this assumption.
   d) Creation of point or eventually range parameter estimations from this reduced set.
2. Create probability model of frequency from model of period lengths.
3. Based on given level of significance $0 < \alpha < 1$ form zones in which common values occur.
4. Further diagnostic comparisons and decisions.

5 Example of process

Observed values:

Inx	Minimum	Average	Sampled median	Maximum	StD. sampled	Skewness - Excel	Kurtosis - Excel	Count	Observed period lengths [sample count]																			
2	164	170	170	180	4,4	0,70	0,11	20	180	173	178	171	174	173	174	170	171	169	170	168	169	165	167	168	165	165	164	166

Tab. 1. Observed period lengths

Values after randomization and reduction by normality tests.

Inx	Minimum	Average	Sampled median	Maximum	StD. sampled	Skewness - Excel	Kurtosis - Excel	Count	Observed period lengths [sample count]																			
2	164	169	169	178	3,7	0,45	-0,23	19		173,13	177,84	170,55	174,08	172,64	174,15	169,92	170,71	168,56	170,48	167,71	169,42	164,91	167,23	168,39	164,95	165,14	163,97	166,31

Tab. 2. Values after randomization and reduction

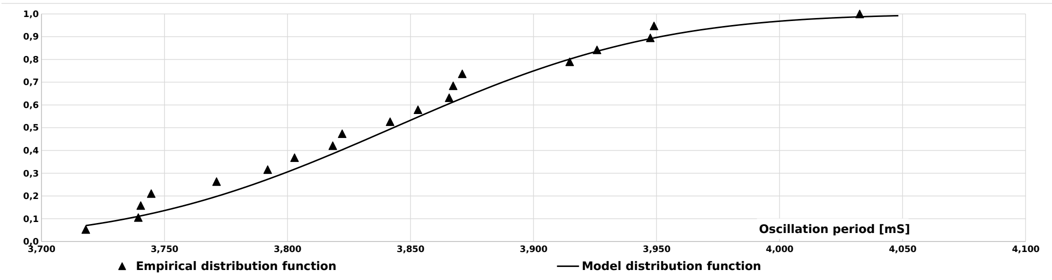

Fig. 2. Comparison of empirical data and model distribution function

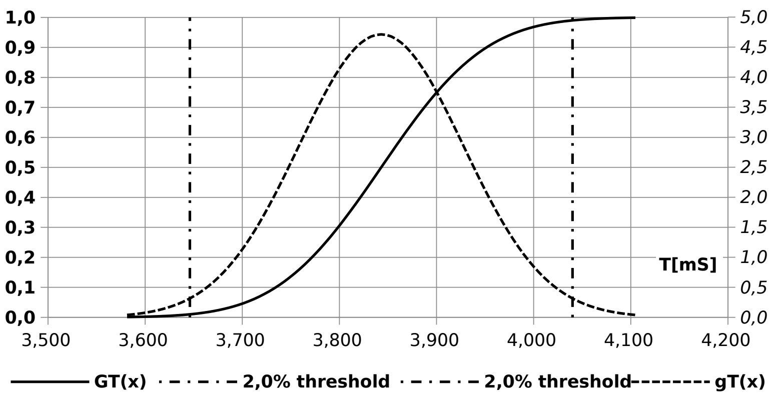

Fig. 3. Modelled distribution functions and density functions for period lengths

Bounds for common or anomaly test and quantile parameters.

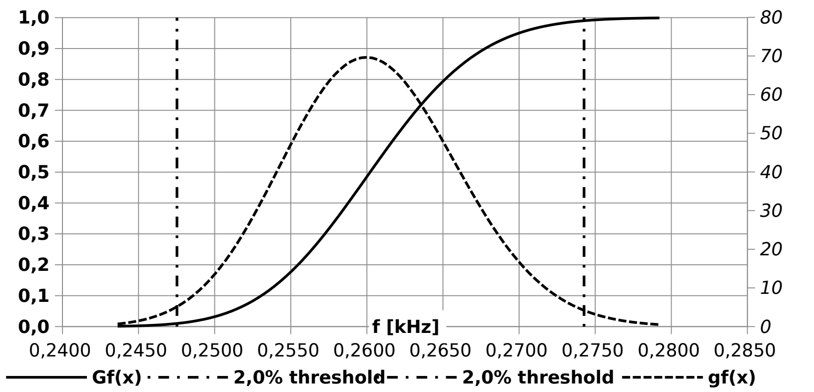

Fig. 4. Modelled distribution functions and density functions for immediate frequencies

6 Conclusion, discussion of results and further work

We assume probability model of oscillation for partial-diagnostic from standardized phonation. Research in this field implies probability model of immediate frequency resemble normal probability distribution, even though it's analytical expression is significantly different. For example density:

$$g_f = \frac{1}{x^2} g_T(\frac{1}{x}) = \frac{1}{x^2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\frac{1}{x}-\mu)^2}{2\sigma^2}} = \frac{1}{x^2} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(1-\mu x)^2}{2(\sigma x)^2}}$$

This holds not only for presented data but for all data available. This leads to following questions:

- Is this the case only for our data, or is it general fact?
- Does it confirm intuitive approaches, when normal distribution is used for both random variables?
- If this is general fact, define scope and how this could be utilized?
- If this is general fact, what is the physiological, physical and acoustical basis?

It is definite description and probability restriction (Bonferroni inequality) for our modified algorithm of normality tests (for example related to Jarque-Bera test) with respect to identification of outlier observations. Parametric and non-parametric descriptions of ideal oscillation shape (in given domain that would be mainly more or less randomly distorted harmonic signals). Degree of "deviation" from estimated ideal shape, a measure with diagnostic value. Joint distribution of triplets period (frequency), shape parameters and degree of deviation.

References

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[6] Jílek, M.: *Statistické toleranční meze*, Book, SNTL, Prague 1988

7 Appendix: Normality test

Based on skewness and kurtosis[5]

Sampled skewness:

$$A_3 = \frac{\sqrt{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{[\sum_{i=1}^{n}(x_i - \overline{x})^2]^{\frac{3}{2}}} - 3$$

with assumption of sample selection from normal distribution is:

$$E(A^3) = 0$$

and

$$\sigma^2(A_3) = \frac{6(n-2)}{(n+1)(n+3)}$$

Sampled kurtosis:

$$A_4 = \frac{n\sum_{i=1}^{n}(x_i - \overline{x})^4}{[\sum_{i=1}^{n}(x_i - \overline{x})^2]^2} - 3$$

with assumption of sample selection from normal distribution is:

$$E(A_4) = -\frac{6}{n-1}$$

and

$$\sigma^2(A_4) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}$$

. Asymptotically has the A_3 distribution[6] $N = (0, \frac{6(n-2)}{(n+1)(n+3)})$ and A_4 distribution $N = (-\frac{6}{n+1}, \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)})$. Therefore the hypothesis of normality on level of confidence α is rejected, if: $\frac{|A_3|}{\sqrt{\sigma^2(A_3)}} > u_{1-\frac{\alpha}{2}}$ or $\frac{|A_4 + \frac{6}{n+1}|}{\sqrt{\sigma^2(A_4)}} > u_{1-\frac{\alpha}{2}}$ If the hypothesis is rejected by at least one of stated tests, observation by which removal will lead to improvement of affected test has to be found, i.e. the test will draw closer to critical value or eventually surpass this value. This

[5] See for example [5], page 138.

[6] Convention $N = (\mu, \sigma^2)$

observation is removed from further statistical processing. This process is repeated until the normality hypothesis is accepted by both tests or until the cardinality of the set decreases under acceptable threshold. This is "brute force" algorithm which repeatedly iterates over whole set. Detailed description of individual steps and decisions exceeds scope of this paper. We expect it would be published including study of premises needed for effective function.

Current address

Lukáš Kroupa, Ing.
Department of Computer Science and Engineering,
Faculty of Applied Sciences, University of West Bohemia
Univerzitní 8, 306 14 Pilsen, Czech Republic
E-mail: kroupal@kiv.zcu.cz

František Vávra, doc. Ing., CSc.
Department of Mathematics,
Faculty of Applied Sciences, University of West Bohemia
Univerzitní 8, 306 14 Pilsen, Czech Republic
Tel. number: +420 377 63 2615, e-mail: vavra@kma.zcu.cz

Pavel Nový, Ing. PhD.
Department of Computer Science and Engineering,
Faculty of Applied Sciences, University of West Bohemia
Univerzitní 8, 306 14 Pilsen, Czech Republic
Tel. number: +420 377 63 2411, e-mail: novyp@kiv.zcu.cz