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DIAGNOSTIC MEANING OF CORRELATION RELATIONSHIP

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Abstract. Brief description of a method based on computing correlation between parameters for vocal cords recorded by high speed video camera. This method gives us possibility to analyze vocal cords kinematics and tries to evaluate specific vocal cords based on statistical data.

Keywords: vocal cords, glottis, correlation analysis, statistics, early diagnosis

Mathematics Subject Classification: Primary 62P10; Secondary 62F10, 62F25

1 Introduction

Highspeed Video technology (HSV) is one of the commonly used vocal cords kinematics examinations these days. The recording of examinations enables to monitor real movement of the vocal cords in the time-phased form. Since our goal is to evaluate the kinematics of vocal cords and it may be insufficient to evaluate it just by visual observation, we are looking for parameters and evaluation techniques to characterize changes in the shape of the vocal gap (glottis) over time during one or more periods and evaluate quality of the closing and opening process. Geometry based parameters derived from the shape of glottis, anatomical axis of the vocal cords and its symmetry makes the group of parameters, see [1], [2], [3].

It seems to be an interesting approach based on the assumption that majority of the geometrical parameters of glottis should have strong "correlation relationship" for their at least locally linear or linearized relation in so-called normal states. From the diagnostic point of view, the situation is significant when "correlation relationship" of some parameters is broken in a video recording of a patient examination, it means values of "correlations" between specific parameters are unexpectedly low. This could be an indicator of the anomalous (pathological) kinematics of the vocal cords.

2 Mathematical basis of vocal cords kinematics evaluation

Identifying (linear or linearizable) relations in statistics often leads to the use of least squares methods. Usually this implies some probability and statistical assumptions (random sample iid, normality of residues, mean value of a random component equals to zero, …). However, these are not required for the applications of various least-squares methods. For our purposes we will revamp the historical least-square methods and also show the relation of point estimates of some probability parameters with estimations of the coefficients of the regression lines by the least squares method.

For a line $y = ax + b$, interleaves data $(x_i, y_i\ ; i = 1, ..., n)$ using the least-squares methods implies, see [4]:

$$a = \frac{\sum_{j=1}^{n}(x_j - x)y_j}{\sum_{j=1}^{n}(x_j - x)^2} \quad \text{and} \quad b = y - ax, \tag{1}$$

where

$$x = \frac{1}{n}\sum_{j=1}^{n} x_j \quad \text{and} \quad y = \frac{1}{n}\sum_{j=1}^{n} y_j\ .$$

After minor modifications and extensions we get the formula:

$$a = \frac{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)(y_j - y)}{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)^2}\ \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - y)^2}} \frac{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - y)^2}}{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)^2}}, \tag{2}$$

where

$$r = \frac{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)(y_j - y)}{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)^2}\ \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - y)^2}} \tag{3}$$

is the same formula like for sample correlation coefficient regardless if the paired data $x_j\ ; j = 1, ..., n$ and $y_j\ ;\ j = 1, ..., n$ are random samples or not.

Under the same conditions are the formulas

$$S_x = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)^2} \quad \text{a} \quad S_y = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - y)^2} \tag{4}$$

numerically equals to the sample standard deviations.

Then also formulas

$a = r\frac{S_y}{S_x}$ and $b = y - r\frac{S_y}{S_x}x$ are numerically identical to expressions for point estimates of regression line coefficients.

These lead to

$$Q(r) = \sum_{i=1}^{n}(y_i - ax_i - b)^2 = (n-1)S_y^2\left(1 - r^2\right), \tag{5}$$

if $r = \pm 1$ is $Q(r) = 0$ and if $r = 0$ is $Q(r) = (n-1)\,S_y^2$.

Quadratic error of linear approximation of data (by least-squares method) has the same properties as the quadratic error of linear regression estimates.

It means for $r \to \pm 1$, quadratic error drops to zero, and for $r \to 0$, quadratic error increases to $(n-1)\,S_y^2$.

Therefore, the expression (3) to which applies

$$r = \frac{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)(y_j - y)}{\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_j - x)^2}\;\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(y_j - y)^2}} = \frac{\sum_{j=1}^{n}(x_j - x)(y_j - y)}{\sqrt{\sum_{j=1}^{n}(x_j - x)^2}\;\sqrt{\sum_{j=1}^{n}(y_j - y)^2}}, \tag{6}$$

is a good measure of "linear force and direction or relation" between *x* and *y*, like the standard correlation coefficient (or its estimate). Because of this it makes sense to call *r* as correlation (although it might not be in the strict sense). It is also possible to use software procedures for linear regression (to the extent described herein).

In total, the line approximating the data $(x_i, y_i\ ; i = 1, \ldots, n)$ by least-squares method in *Y* axis is given by formula:

$y = r\frac{S_y}{S_x}x + \left(y - r\frac{S_y}{S_x}x\right)$ and its total quadratic error is $Q(r) = (n-1)\,S_y^2\,\left(1 - r^2\right)$.

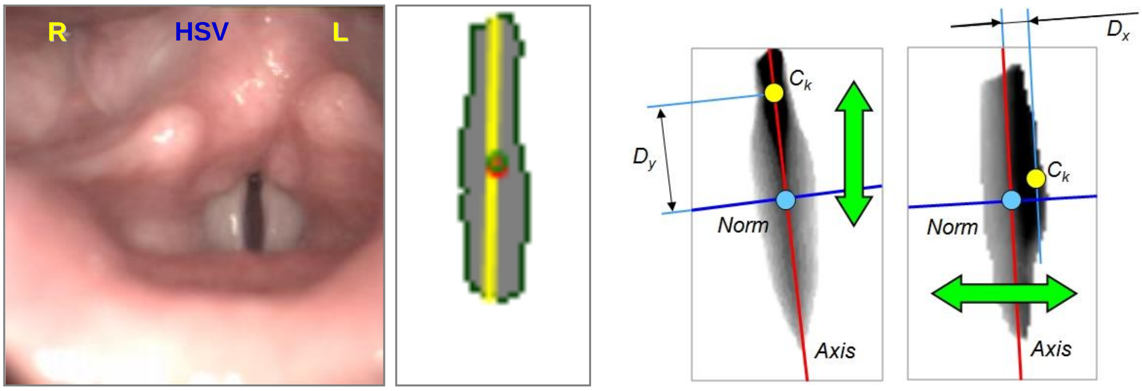
Similarly, it is possible to derive formulas for a line approximating the data $(x_i, y_i\ ; i = 1, \ldots, n)$ by least-square method in the *X* axis as

$x = r\frac{S_x}{S_y}y + \left(x - r\frac{S_x}{S_y}y\right)$ and its total quadratic error as $Q(r) = (n-1)\,S_x^2\,\left(1 - r^2\right)$.

Since expressions $\sqrt{S_x^2}$ and $\sqrt{S_y^2}$ determine the scale on the *X* and *Y* axes, the quality (measured by the summary quadratic error *SSE*) of the approximating line is given by the expression $\left(1 - r^2\right)$.

3 Application for vocal cords kinematics evaluation

From the set of parameters evaluating the properties of the vocal cords, see [2], [3], parameters based on the geometry of the vocal cords, especially the geometry of the glottis, were used to study the “correlation relationship”.



(a) (b) (c) (d)

Fig. 1. Explaining picture of geometric parameters of glottis

a) Picture of the vocal cords taken by HSV (High Speed Video)
b) Segmented glottis from the HSV with highlighted inner perimeter, axis of the symmetry, center point of area and inner perimeter of the glottis
c) Scheme of movement of the location D_x, D_y of center point C_k along the symmetry axis *Axis* and along normal line *Norm* during one period of symmetrical vocal cords
d) Scheme of movement of the location D_x, D_y of center point C_k along the symmetry axis *Axis* and along normal line *Norm* during one period of non-symmetrical vocal cords

The group consists of 14 selected geometrical or geometric derivable parameters arranged in 91 possible pairs to compute mutual correlations, see Fig. 1 and Tab 1.

	parameter	*variable*
1	Size of area of glottis	A
2	Size of left part of glottis area (left from symmetry axis)	A_{left}
3	Size of right part of glottis area (right from symmetry axis)	A_{right}
4	Deflection of the area as difference between left and right area parts	A_{diff}
5	Deflection of center point of the area from symmetry axis	D_xS
6	Deflection of center point of the perimeter from symmetry axis	D_xH
7	Difference between area and perimeter center point in the normal line direction	$D_x\,diff$
8	Deflection of center point of the area from the normal line	D_yS
9	Deflection of center point of the perimeter from the normal line	D_yH
10	Difference between area and perimeter center point in the symmetry axis direction	$D_y\,diff$
11	Length of glottis perimeter ≈ inner perimeter of glottis	P
12	Length of left side of glottis perimeter (left from symmetry axis)	P_{left}
13	Length of right side of glottis perimeter (right from symmetry axis)	P_{right}
14	Difference between left and right parts of the perimeter	P_{diff}

Tab. 1. Table of selected geometric parameters of glottis

According to that, there should be a strong relationship in standard conditions for these parameters. These relationships should then be mostly linear or well linearizable. Such trivial relationship is e.g. relationship between area A and left/right part of the area A_{left}/A_{right}, see Fig. 2 and Fig. 3. This will show the advantage of the “correlation relationship” which is independent on the scale. In case of Fig. 2 it is a healthy symmetrical vocal cords.

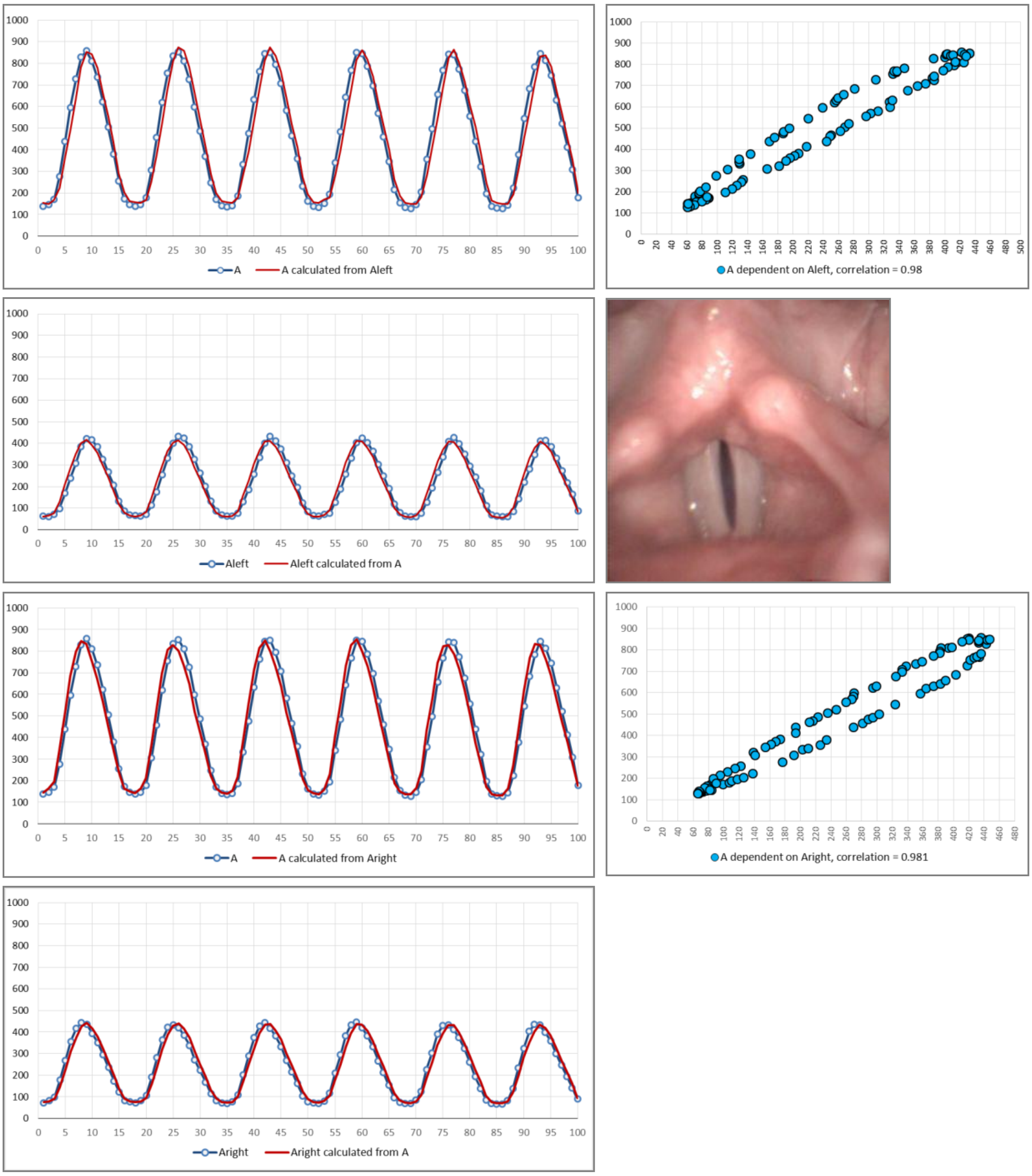


Fig. 2. Demonstration of correlation relations for healthy vocal cords of parameter pair A/A_{left}, correlation value = 0,980, and parameter pair A/A_{right}, correlation value= 0,981

Diagnostic meaning is expected at low "correlation relations" of such parameter pairs. As an example (casuistic), vocal cords with paresis of the left reversible nerve after an injury is presented, see Fig. 3, Fig. 4 and Fig. 5. Because of low movability of left vocal fold, "correlation relationship" is significantly broken between parameters A/A_{left}, strong "correlation relationship" between parameters A/A_{right} is kept, see Fig. 3.

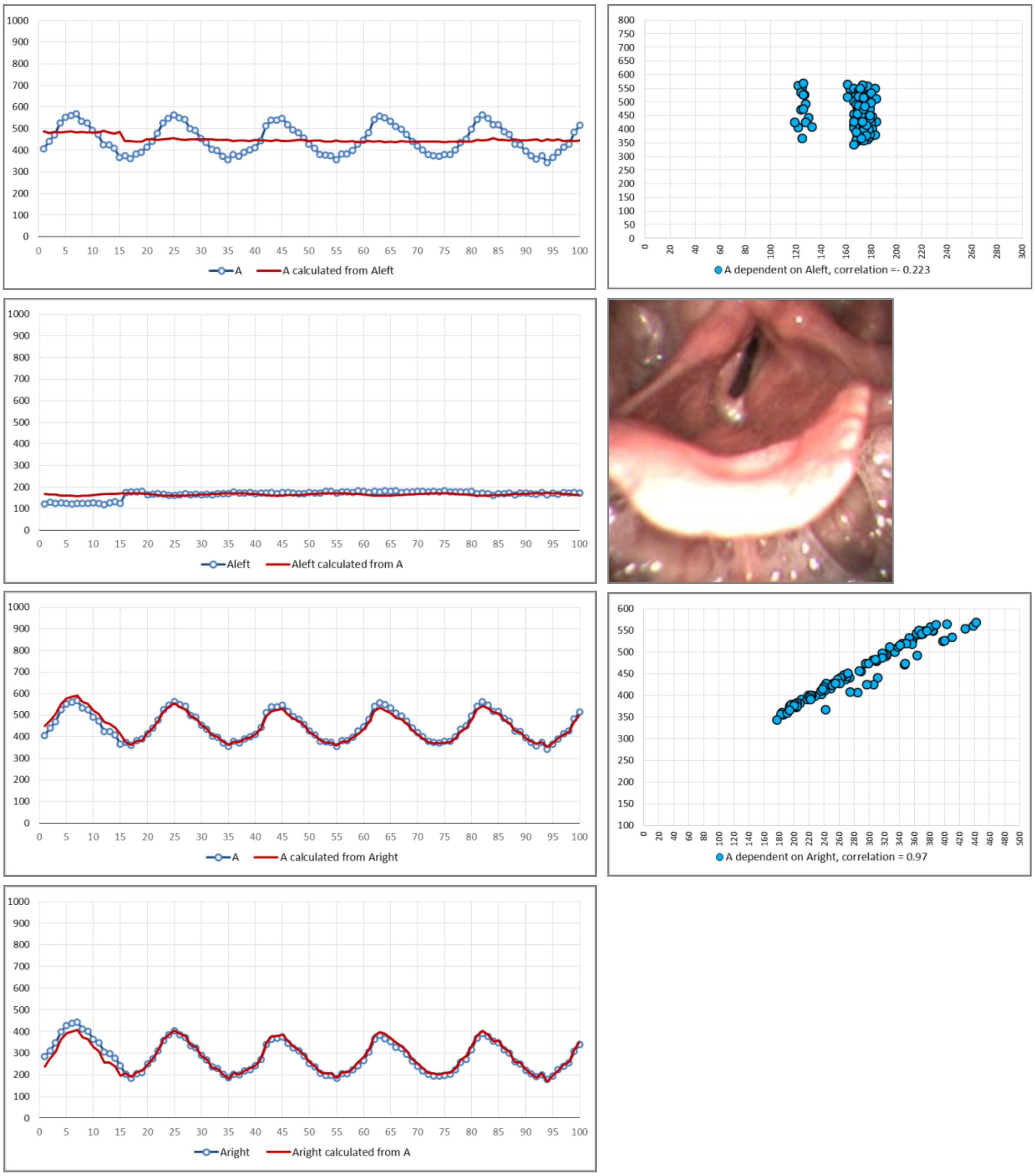


Fig. 3. Demonstration of correlation relations for non-healthy vocal cord (dg. single sided paresis on the left side) for parameter pair A/A_{left}, correlation value = 0,223, and parameter pair A/A_{right}, correlation value = 0,970

Dynamic properties of left vocal fold (small and random movements) also influences the “correlation relationship” between locations of center points D_yS/D_yH in the direction of symmetry axis, see Fig 4. The progress of area and perimeter center points’ location in the normal line direction D_xS and D_xH demonstrates regular movement caused by significant oscillation of the right vocal fold, see Fig 5.

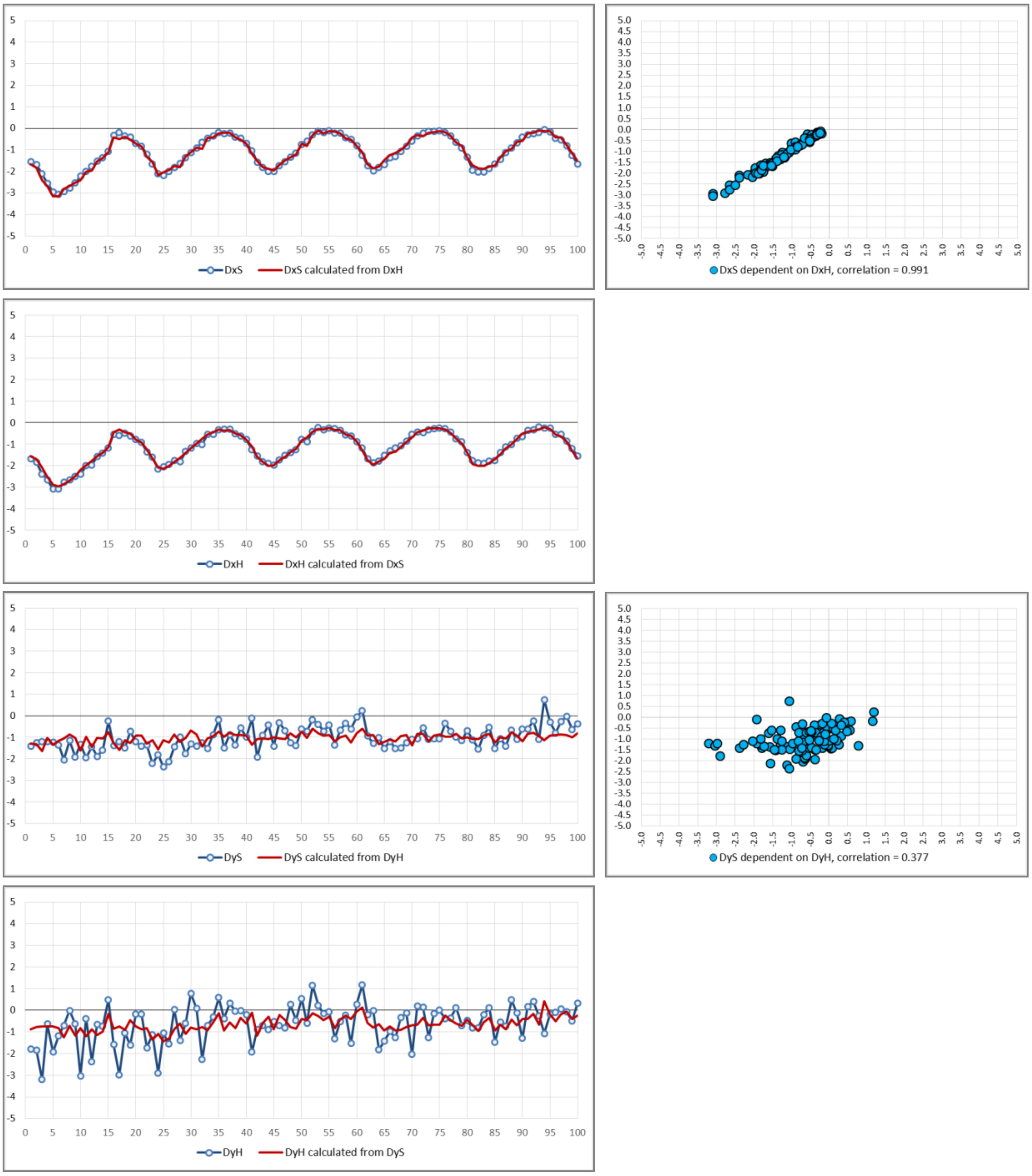


Fig. 4. Demonstration of correlation relations for non-healthy vocal cord (dg. single sided paresis on the left side), for parameter pair D_xS/D_xH, correlation value = 0,991, and parameter pair D_yS/D_yH, correlation value = 0,377

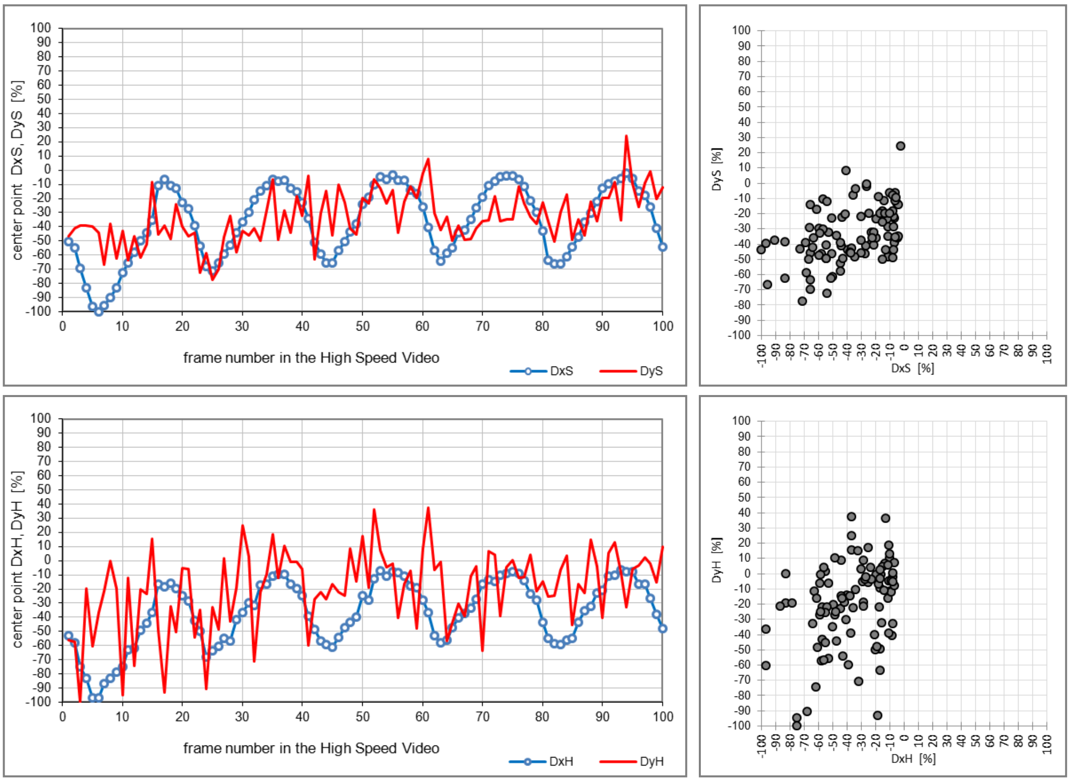


Fig. 5. A movement of the area center point D_xS and D_yS and a movement of the perimeter center point D_xH and D_yH for non-healthy vocal cord (dg. single sided paresis on the left side). Single sided deflection of the center point is observable.

4 Summary

The relationships between 91 possible pairs of geometric parameters from more than 400 high speed video recordings were analyzed, each recording contains at least 100 frames.

For the simplest decision-making process, just select the following four limits:

- Lower limit to indicate *SIGNIFICANT* = common correlations:
  Threshold 0.894 was chosen, it corresponds to determination coefficient 0.800
  (it means if we estimate one of the pair parameter through the other, we reduce its own variability by at least 80 %).
- Upper limit to indicate *EXCEPTIONAL* correlations:
  Threshold 0.447 was chosen, it corresponds to determination coefficient 0.200
  (it means if we estimate one of the pair parameter through the other, we can reduce its own variability by at most 20 %).

Common and exceptional data are given by the frequency of occurrence in a given population, therefore:

- Lower frequency limit for marking *SIGNIFICANT* = common correlations:
  Threshold 66.3 was chosen
  (at least 2/3 of records of given parameter pair indicate at least *SIGNIFICANT* correlation).
- Upper frequency limit for marking *EXCEPTIONAL* correlations:
  Threshold 10.0 % was chosen
  (at most 10 % of records of given parameter pair indicates *EXCEPTIONAL* correlation.

From there, we will find parameter pairs for possible marking of *EXCEPTIONAL* records: [A/A_{left}], [A/A_{right}], [A/P], [DxS/DxH], [DyS/DyH].

Of course it is possible to analyze correlation structure by more sophisticated methods. It was also done, however, its description goes far beyond the scope of this paper. The version presented was chosen for illustration and clarity.

In addition, it should be noted that this methodology is applicable in all cases where failure of the relationship of geometric parameters can lead to diagnostic significance.

Acknowledgements

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References

[1] ETTLER, T., *Detection and Evalution of Glottis in High Speed Video Recording*, Professional work for the state doctoral exam, University of West Bohemia, Pilsen, 2017.

[2] ETTLER, T., NOVÝ, P., *The parameters of the Center of gravity glottis during phonation (Parametry polohy těžiště hlasivkové štěrbiny průběhu fonace)* XVI. Česko-slovenský kongres mladých otorinolaryngologů, poster section (e-poster), ISBN 978-80-87562-57-4, Česká lékařská společnost Jana Evangelisty Purkyně, Rožnov pod Radhoštěm, 2016.

[3] PEŠTA, J., SLÍPKA, J., VOHLÍDKOVÁ. M., ETTLER. T., NOVÝ, P., VÁVRA, F., *Vocal Cord Kinematics - New Evaluation Parameters (Kinematika hlasivek - nové parametry hodnocení)*. Otorinolaryngologie a foniatrie, Vol. 65, Issue 2, pp. 88-96, ISSN 1210-7867, Česká lékařská společnost Jana Evangelisty Purkyně, Praha, 2016.

[4] HÁTLE, J., LIKEŠ, J., *Základy počtu pravděpodobnosti a matematické statistiky*. SNTL, Praha, 1974.Current address

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