INTRODUCTION

The meaning and characteristics of trust were described in many works focused on social issues (Fukuyama 1995; Seligman 1997; Sztompka 1999; Gambetta 2000). In next steps trust models were created (Chang et al. 2005; Lifen 2008). Further, trust plays an important role in e-service (Liu et al. 2008), e-commerce (Zhang et al. 2008), e-banking, peer-to-peer networks (Chen and Yeager 2008). Wide-spreading of communication through social networks, where humans have to collaborate, asked adding trust to these electronic systems (Mui 2002; Carrington et al. 2005; Dwyer et al. 2007; Richters et al. 2011).

Various approaches were designed for computational models of trust. Let us give just a few examples based on agent technology (Rettinger et al. 2007; Sankaranarayanan et al. 2010a), fuzzy systems (Chang et al. 2005; Sankaranarayanan et al. 2010b), Markov models (Hussain et al. 2005), or game theory (Sankaranarayanan 2007).

The aim of our work is building trust model and simulation of the trust evolution in social network. The model closely reflects members of social networks, especially introduces differentiating members in their disposition to trust somebody.

The organization of the paper is following. Firstly, the term trust and its representation are introduced. Next, we introduce the trust model and we use it for trust affection modeling in social network. Finally, behavior of the trust model is investigated and the results are presented.
The reputation of the subject comes after the individuals that are by the individuals who have passed on it is called recommendation. The reputation of the subject is considered factors, which are reputation and attitude comprises more values—believe or do not believe. The reputation of the subject is formed from three actors, called triad, is in Figure 3. Shaded line represents orientation of both trust, and reputation. This convention will be used in Figures in the rest of the paper as well.

In our model, the relationships among the actors $s_i (i=A, B, C)$ in triad are represented by reciprocal trust $t_{ij}$ and $t_{ji}$ ($i, j = A, B, C, \ i \neq j$), reputation $r_{ij}$ and $r_{ji}$, and trusting disposition $g_{ij}$. Sending the recommendation to actor modifies its trust is described in detail in next section. A simple example of recommendation $d^{ac}_{ij}$ which is sent from actor $C$ to actor $B$ about its trust to actor $A$ is shown in Figure 4.

Based on the recommendation (dotted line in Figure 4) trust $t_{rA}$ and reputation $r_{rA}$ (dashed line in Figure 4) will be modified.

### Social Network Trust Model Description

Trust formation $T_{ij}$ between two actors (trustor and trustee) is generally expressed as function

$$T_{ij} = f(t_{ij}, d^{ij}_{ij}, r_i, g_i), \quad (2)$$

where $t_{ij}, r_i$ is reciprocal trust of actors, $d^{ij}_{ij}$ are the recommendations of $i$-th actor to $j$-th actor about trust to $k$-th actor, $r_i$ is reputation of $j$-th actor, and $g_i$ is trusting disposition of $i$-th actor.

Reputation of an actor is computed from its neighbours, which provide their rating of the actor. The rating scale has six degrees—very negative experience VNE, negative

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**Figure 2: Trust in Group**

Interpersonal trust is formed by many factors. Besides usually considered factors, which are reputation and recommendation, we introduce a subject’s trusting disposition. The reputation of the subject comes after individual experience and by some information dissemination about subject in its neighbourhood and influences trust formation considerable. Information about another subject that other subjects have passed on is called recommendation. Trusting disposition represents a non rational behaviour of a subject and is modeled by random factor.

**TRUST IN SOCIAL NETWORK**

Social network can be described as a social structure created by the individuals that are bonded together on the basis of some particularity. This particularity can cover e.g. family relationship, friendship, financial transactions, common interest, and so on.

Applications of social networks are used in biology for disease spread simulation, in economy and marketing to secure higher profit, e.g. advertisements aimed at population groups. Internet and information technologies have the most significant role in social network applications. Social network is not only global network Facebook or Twitter, but also auction portal e-Bay, and its Czech modification Aukro, or specialized servers, e.g. Czech server Heureka focused on discussions about experience and practice in e-commerce. Social network connects individuals in the groups. Individuals in social network are called actors. Relationships among actors form attitude of one actor to another one. The simplest attitude is binary one—believe or do not believe; in reality the attitude comprises more values. Primary factor in the process of forming attitude is trust.

Generally, social networks are modelled by the oriented weighted graphs similarly to the personal trust model.

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**Figure 3: Trust Relations in Triad**

**Figure 4: Trust Relations with Recommendation**
experience NE, neutral - negative NN, neutral positive NP, positive experience PE and very positive experience VPE. For each degree, a real value is assigned (see in Table 1). The reputation of an actor is arithmetic mean of the actor’s neighbours rating values.

Table 1: Rating, Degree and Value Description of Reputation

<table>
<thead>
<tr>
<th>Rating</th>
<th>Degree</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VNE</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NE</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>NN</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>NP</td>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>PE</td>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>VPE</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Actual trust value \( T_{ij} \) is produced by two components - previous trust \( t_{ij} \) and change of trust (gain or loss)

\[
T_{ij} = t_{ij} + \Delta t_{ij}. \tag{3}
\]

Thus, for trust variation of i-th actor (trustor) to j-th actor (trustee) the formula (4) is proposed

\[
\Delta T_{ij} = \sqrt[3]{s_{ij}}d_i^1 w_j r_j w_i \tag{4}
\]

where \( t_{ij} \) is previous trust of i-th actor in j-th one, \( t_{ij} \) is previous trust of j-th actor in i-th one, and tendency of reciprocal trust influence is reflected by geometric mean;
\( d_i \) is average of recommendations about j-th actor to i-th computed by formula

\[
d_i = \sum t_{ik} t_{kj} \frac{p}{i}, \tag{5}
\]

where \( t_{ik} t_{kj} \) exists, and \( p \) is number of those actors;
\( r_j \) is reputation of j-th subject described by formula

\[
r_j = \frac{\sum r_{ij}}{q}, \tag{6}
\]

where \( r_{ij} \) is actor’s rating values given by \( q \) actors in its neighborhood; \( g_i \) is trusting disposition expressed by the probability distribution function; influence of recommendation, reputation, and trusting disposition is determined by the weight coefficients \( w_d \), \( w_r \), and \( w_e \) from the interval (0, 1).

Then, final trust \( T_{ij} \) formula is following

\[
T_{ij} = t_{ij} + \sqrt[3]{s_{ij}} \left( \frac{\sum t_{ik} t_{kj}}{p} \right) w_d \left( \frac{\sum r_{ij}}{q} \right) w_r g_i w_e \tag{7}
\]

CASE STUDY

To illustrate trust evolution under parameter changes, we took an example of small social network on which the essential cases of trust changes are shown (Havel 2011).

Social network consists of fourteen actors with their trust relations. Actors and their known contacts in social network are given by the matrix of actors \( S \). Matrix entries \( S_{ij} \) is 1 when the actors \( i,j \) know each other, and \( S_{ij} = 0 \) otherwise.

Trust matrix \( T \) describes reciprocal trust of actors. Existing trust is given by value in the interval (0, 1). Value -1 represents the situation when the actors do not know each other or the fact that reciprocal trust is not known.

\[
T = \begin{bmatrix}
-1 & 0.2 & 0.99 & -1 & -1 & 0.99 & -1 & 0.99 & -1 & 0.99 & -1 & 0.6 & 0.8 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
0.99 & 0.99 & -1 & -1 & 0.4 & 0.9 & 0.9 & 0.7 & 0.6 & 0.3 & 0.3 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0.2 & 0.01 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
0.99 & 0.99 & 0.9 & -1 & -1 & 0.6 & 0.2 & -1 & -1 & -1 & -1 & 0.7 & -1 & -1 & -1 \\
-1 & -1 & -1 & 0.4 & -1 & -1 & 0.8 & -1 & -1 & -1 & -1 & -1 & -1 & 0.8 & -1 & -1 \\
0.8 & -1 & -1 & -1 & -1 & 0.3 & -1 & -1 & -1 & -1 & 0.3 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0.7 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0.4 & 0.01 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0.6 & -1 & 0.7 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
0.5 & -1 & 0.3 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0.8 & -1 & -1 & 0.2 & -1 & -1 \\
0.4 & -1 & -1 & -1 & -1 & -1 & 0.1 & 0.01 & -1 & 0.7 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 0.8 & -1 & -1 & -1 & -1 & 0.8 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & 0.4 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
\end{bmatrix}
\]

Actor rating degrees are placed in the rating matrix \( R \).

\[
R = \begin{bmatrix}
-1 & 0.5 & 0.5 & -1 & 0.5 & -1 & 0.5 & -1 & 0.5 & -1 & 0.5 & -1 & 0.5 & -1 \\
-1 & -1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0 & -1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 0 & -1 & -1 & -1 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
5 & 0 & 3 & -1 & -1 & 1 & 4 & 4 & -1 & -1 & 3 & -1 & 1 & 4 & -1 & -1 \\
-1 & -1 & 1 & 3 & -1 & 2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 4 & -1 & -1 \\
-1 & -1 & 1 & 3 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 3 & -1 \\
-1 & -1 & 0 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
5 & 1 & 5 & -1 & -1 & -1 & 5 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
4 & -1 & -1 & -1 & -1 & -1 & 3 & 5 & -1 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 3 & -1 & -1 & 1 & 5 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 3 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
\end{bmatrix}
\]

Trust dispositions of actors are stored in trusting disposition vector \( G \).

\[
G = \begin{bmatrix}
0.99 & 0.99 & 0.8 & 0.01 & 0.01 & 0.01 & 0.7 & 0.4 & 0.6 & 0.3 & 0.5 & 0.8 & 0.99 & 0.01 & 0.4 \\
\end{bmatrix}
\]

The greater is value of trusting disposition parameter, the higher is tendency to trust anticipation.

The weights - recommendation weight \( w_r \), reputation weight \( w_r \), and trusting disposition weight \( w_e \) were set to the value 0.6 in this case.

Reciprocal trust growth

Let us start with a pair of actors with rather low reciprocal trust. On the other hand they have high reputation and high trusting disposition. One of them has high recommendations,
while the other one’s recommendation is average. Situation is shown in Figure 5. Four actors, i.e. 1, 2, 3 and 6, create the core of chosen part of social network. Pair of actors under study is 1, 2. We expect subsequent growth of both trust $t_{12}$, $t_{21}$, while the first should grow more rapidly. The results from our model depicted in Figure 6 are in good concordance with the expectation.

Reputation values of actor 11 gradually increase from 0 to 5 (rating matrix $R$, row 11). The other parameters do not change. Trust growth between actor 3 and actor 11 is depicted in Figure 8 and is in good in concordance with expected behavior.

**Recommendation influence**

In this study the influence of recommendations on trust evolution is explored. Recommendation can be sent to direct neighbor actor only about direct neighbor of sender actor. To demonstrate the influence we selected the subgraph with actors 1, 6, and 8 (see Figure 9).

Studied trust is between actor 6 and actor 8. The growth of this trust is brokered by common friend - actor 1 that will send the recommendation about actor 8 to actor 6. Influence of recommendation depends on trusts of actor 6 into actor 1, and actor 1 into actor 8. The value of element of trust matrix $T[6,1]$ is gradually changed to 0.1, 0.3, 0.5, 0.8, and 0.99, in association with element of trust matrix $T[1,8]$ which value is changed to 0.2, 0.3, 0.6, 0.8, and 0.99 pair wise.

The results of this study are shown in Figure 10. According to expectation trust increases more rapidly with higher recommendations, cause by increase of trust between broker-actor and recommended actor as well between recommending actor and broker.
CONCLUSION

We developed the personal trust representation, where trust values are from continuous interval compared e.g. to Markov models using trust levels. Based on this representation, trust model for social networks was developed and implemented. Model enables to study the dynamics of trust evolution in a social network under changing trust forming factors. Upcoming model modification will allow covering the effect of intentional trust affection of social network members.

REFERENCES


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ARNOSTKA NETRVALOVA was born in Plzen, Czech Republic. She is senior lecturer in Department of Computer Science and Engineering at Faculty of Applied Sciences of University of West Bohemia. She holds M.Sc. in Computer Science from University of West Bohemia in 1977. Her research interest in modeling and simulation in medicine covered models of temperature homeostasis. She received Ph.D. from the same university in 2010. Her present research is focused on trust modeling and simulation.

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