Independent Component Analysis

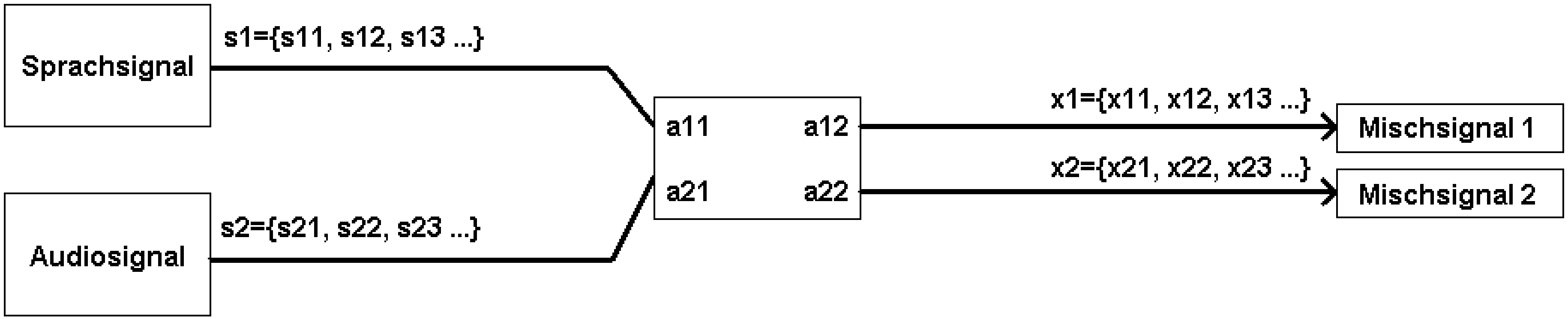
Problem

- Medicine
  - – Division of EEG-Signals
  - – NMR image processing
- Analysis of faces
- Radio communication
- Audio-de-mixing

„Cocktailparty-Problem“

- Situation:
  - 2 talkers
  - Loud music
- Problem:
  - Understanding each other

„Cocktailparty-Problem“



s = (s1 ; s2)
x = (x1 ; x2)
A = (a11 a12 ; a21 a22)

x = A•s

„Cocktailparty-Problem“

- De-mixing
  - Mixing: x = A*s
  - De-Mixing:

    u = W*x,

    if W = inv(A)

    then u = s

- Problem
  - A unknown

„Cocktailparty-Problem“

- Solution
  - Find W with probabilistic / information-theory-based approach

Independent Components ?

- Already known:

  independency of two signals X,Y means:

  $P(X,Y) = P(X)*P(Y)$

Information Theory

- Information:
  - Measure of „surprise“ about an unobserved value
  - Definition (Shannon) : „One Bit is the Information of an event with probability of occurence = 0.5“

$$I(X) = \sum_i \log_2 \frac{1}{P(X_i)}$$

Information Theory (II)

- Entropy
  - Measure of disorder in a system
  - Equivalent to maximum information
  - For discrete case:

$$H(X) = \sum_i P(X_i) \cdot \log_2 \frac{1}{P(X_i)}$$

Information Theory (III)

- Joint entropy

$$H(X,Y)=\sum_i P(X_i,Y_i)\cdot\log_2\frac{1}{P(X_i,Y_i)}$$

- Conditional Entropy

$$H(X/Y)=\sum_i P(X_i,Y_i)\cdot\log_2\frac{1}{P(X_i/Y_i)}$$

- Independent Data X and Y:

$$H(X,Y) = H(X) + H(Y)$$

Information Theory (IV)

- Transinformation I
  - Measure for the information that X contains about Y (and vice versa)
  - $I(X,Y) = H(X) + H(Y) - H(X,Y)$
  - If X and Y independent: $I(X,Y) = 0$

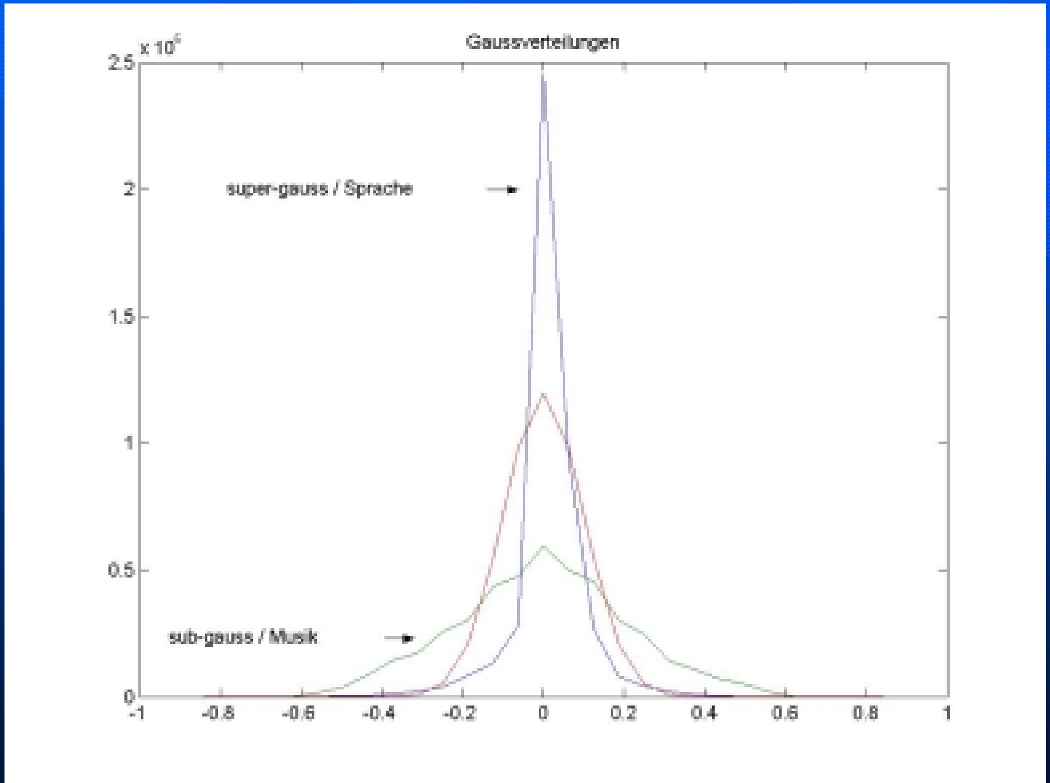
Information Theory (V)

- Kullback-Leibler-Divergence
  - Measure for the „distance“ between two probability distributions P(x), Q(x)
  - Zero, if P(x) = Q(x)

$$\mathrm{KL}(P \,||\, Q) \equiv \sum_x P(x) \log \frac{P(x)}{Q(x)}$$

Information Theory (VI)

- Normal and related distributions:



„Fast-ICA“-Algorithm

- Realisation:
  - One-layer Neural Network with weight vector W
  - Non-linear decision function g(u)
  - „feedforward“ with natural gradient
- Optimization of Neural Net through Maximization of total entropy
  - optimize g
  - Optimize W

„Fast-ICA"-Algorithm (II)

- Optimize nonlinear decision function g
  - Must have:

$$g(u) = \int p(u)\,du$$

„Fast-ICA“-Algorithmus (III)

- Optimize weights W

$$\frac{\partial H(y)}{\partial W} = \left(W^T\right)^{-1} + \left(\frac{\frac{\partial p(u)}{\partial u}}{p(u)}\right) \cdot x^T$$

„Fast-ICA"-Algorithmus (IV)

- Implementation-optimized learning rule:

$$\Delta W \propto \left[I - \varphi(u) \cdot u^T \right] \cdot W$$

- with:

$$\varphi(u) = u - \tanh(u) \qquad \text{sub-gauss}$$

$$\varphi(u) = u + \tanh(u) \qquad \text{super-gauss}$$

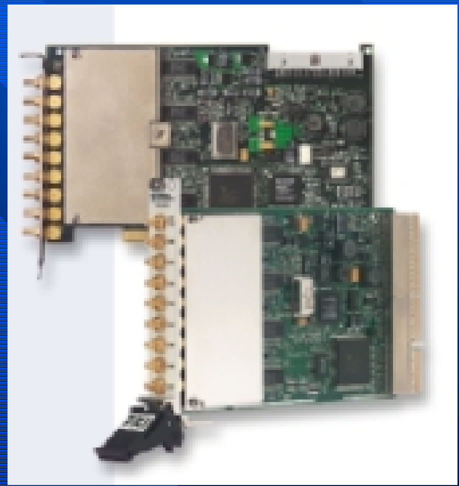
$$\varphi(u) = u + u \cdot \exp\left(\frac{-u^2}{2} \right) \qquad \text{more robust super-gauss}$$

„Fast-ICA"-Algorithmus (V)

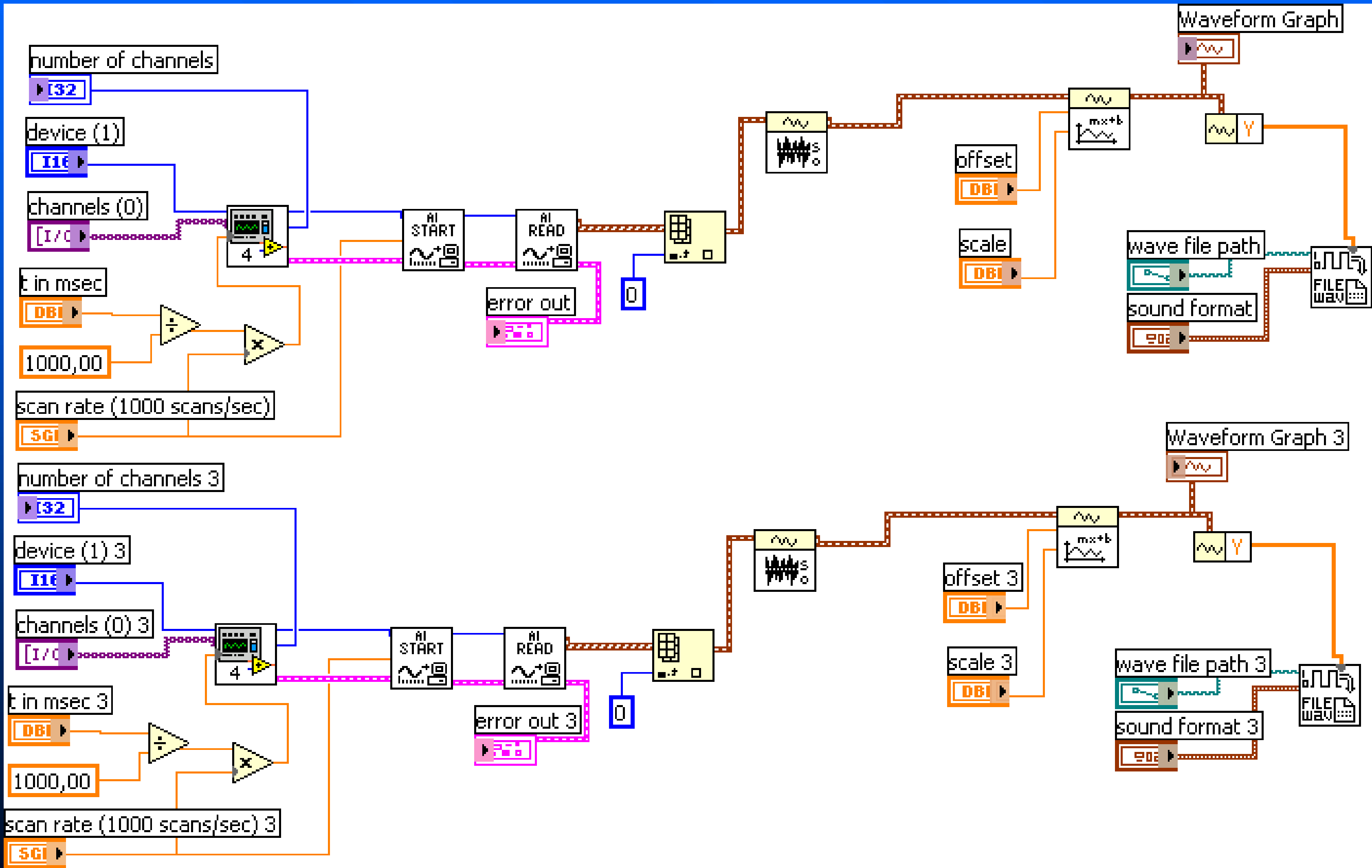
- Implementation in Matlab:
  – Distribution function of the source signal
  – Nonlinear decision function
  – Result (example):

Using Labview

- Hardware in lab:
  - PXI-Industry-PC
  - 8 channel ADU-board for signal capture



Using Labview



Using Labview

- Problems:
  - Model not ideal for recorded signals:
    - » Reflekcions in space
    - » Time delays

Mikro1.wav

Mikro2.wav

Toolbox Fast-ICA

HELSINKI UNIVERSITY OF TECHNOLOGY
Laboratory of Computer and Information Science
Neural Networks Research Centre

- Toolbox for Matlab
  - Link: http://www.cis.hut.fi/projects/ica/fastica/index.shtml

Possible improvements

- Other ICA-Methods
  - Noisy ICA
  - ICA for low numbers of mixed signals („partially blind“), #s > #x
  - Non-linear mixtures in ICA
  - ICA with usage of time structure (Separation through correlations (co-variances)) – works only for signals which obey statistics of order > 2.

Sources

- Literature:
  - Independent Component Analysis – Theory and Applications (Te-Won Lee)
  - Independent Component Analysis – Principles and Practice (Stephen Roberts / Richard Everson)
  - Independent Component Analysis (Aapo Hyvärinen / Juha Karhunen / Erkki Oja)
- Internet:
  - http://www.cis.hut.fi/projects/ica