Department of Mathematics
Faculty of Mechanical Engineering
Slovak University of Technology in Bratislava

2nd International Conference
APLIMAT 2003

APPLICATION OF INSURANCE METHODS IN POWER ENGINEERING

KOTLÍKOVÁ Michala (ČR), MAŠKOVÁ Hana, (ČR), NETRVALOVÁ Arnoštka, (ČR), NOVÝ Pavel, (ČR), VÁVRA František, (ČR), ZMRHAL David, (ČR)

Abstract. Our paper deals with the application of insurance methods in technical disciplines, mainly in power engineering and information technologies. Models used for the description of the cumulative claim are analogous to the probability description of the cumulative power equipment outage duration. The same might be said about the cumulative costs of failure events clearance, their causes and consequences. We focus on the computational methods for model probabilities and we also present statistical methods of the estimations of model parameters. Our methodology comes from the new-fashioned approach to reliability theory where we leave the classical approach working with the failure rate curve, because it has not suitable apparatus for the description of several failure effects (outage duration, costs of failure causes and consequences clearance). We stem from the following trivial equivalences: (insurance event ≡ failure), (insurance claim ≡ outage duration), (insurance claim ≡ costs of failure causes and consequences clearance). The counting process of failure events number and the compound point process of failure effects become grounds for our modelling.

Key words. insurance methods, outage duration, failure, cumulative costs

1 Introduction

In the modelling and prediction of failures and their consequences (power equipment outages), we work with the following processes (random sequences):

Time point process of outages starts:

$$T_1, T_2, T_3, \dots, T_{i-1}, T_i, T_{i+1}, \dots \tag{1}$$

Counting process of the number of outages (failures) occurred up to time t:

$$N(t) = \sum_{i=0}^{\infty} \chi(T_i \le t)\text{, where } \chi(T_i \le t) \text{ is the characteristic function of interval } T_i \le t . \tag{2}$$

Time point process of outage durations:

$$\delta_1, \delta_2, \delta_3, ..., \delta_{i-1}, \delta_i, \delta_{i+1}, ... \quad (3)$$

Time point process of costs of failure causes and consequences clearance:

$$c_1, c_2, c_3, ..., c_{i-1}, c_i, c_{i+1}, ... \quad (4)$$

Compound process of total cumulative costs of the clearance of all failure causes and consequences up to time point *t*:

$$C(t) = \sum_{i=0}^{\infty} \chi(T_i \leq t)\, c_i \quad (5)$$

it can be written also as: $C(t) = \sum_{i=1}^{N(t)} c_i$.

2 Time measurement and other assumptions

In the classical model (5) there is an explicit assumption that the time required for the insurance claim execution is neglecting toward the time period between insurance events. From the technical point of view: the power equipment outage durations are neglecting toward the time periods between failures occurrence. If this assumption can not be verified (and we focus on such situation) it is necessary to introduce some modifications of the time axis.

Modification for the classical model (5): Let's have

$$\tau_1, \tau_2, \tau_3, ..., \tau_{i-1}, \tau_i, \tau_{i+1}, ... ,$$

where τ_1 is the time interval between the device installation and the beginning of the first outage and τ_i is the time interval between the end of *(i -1)*th outage and the beginning of *i*th outage.

Furthermore let:

$$\delta_1, \delta_2, \delta_3, ..., \delta_{i-1}, \delta_i, \delta_{i+1}, ...$$

be the corresponding outage durations. To be able to use the classical model we need to consider two abstract clocks – operational clock and outage clock. Operational clock is switched on when the device is installed and stopped whenever a failure occurs. After the failure clearance (the end of an outage) operational clock is switched on again. Outage clock is switched on within the start of an outage and stopped within its end. Those two abstract clocks are therefore alternating. It is obvious that the time in classical model (5) is the time of operational clock. Time axis holds the following shape (expressed in lengths of the time intervals):

$$\tau_1, \delta_1, \tau_2, \delta_2, \ldots, \tau_i, \delta_i, \tau_{i+1}, \delta_{i+1}, \ldots \quad (6)$$

We assume for observed random variables:

$\tau_1, \tau_2, \tau_3, ..., \tau_{i-1}, \tau_i, \tau_{i+1}, ...$ are independent and identically distributed random variables,

$\delta_1, \delta_2, \delta_3, ..., \delta_{i-1}, \delta_i, \delta_{i+1}, ...$ are independent and identically distributed random variables,

$c_1, c_2, c_3, ..., c_{i-1}, c_i, c_{i+1}, ...$ are independent and identically distributed random variables,

N(t) a $\tau_1, \tau_2, \tau_3, ..., \tau_{i-1}, \tau_i, \tau_{i+1}, ...$ are independent random variables,

N(t) a $c_1, c_2, c_3, ..., c_{i-1}, c_i, c_{i+1}, ...$ are independent random variables,

N(t) a $\delta_1, \delta_2, \delta_3, ..., \delta_{i-1}, \delta_i, \delta_{i+1}, ...$ are independent random variables.

Those are the classical assumptions which enable us to obtain reasonably closed model forms.

3 Classical model

Using the assumptions mentioned above we will observe two cumulative processes. The process of total (cumulative) outage duration *D(t)* up to time point *t* and the process of total (cumulative) causes and consequences clearance *C(t)* again up to time point *t*:

$$D(t) = \sum_{i=1}^{N(t)-1} \delta_i \quad \text{and} \tag{7}$$

$$C(t) = \sum_{i=1}^{N(t)} c_i \, . \tag{8}$$

With respect to the formal similarity of both processes (the usage of operational clock), we will further deal only with the process *C(t)*. The results we obtain will be applicable for the process *D(t)* as well.

Basic apparatus of our model is based on distribution functions:

$$F_C(x,t) = P\{C(\mathrm{t}) \le \mathrm{x}\}\text{, thereinafter denoted as: } F_n(x) = P\{\sum_{i=1}^{n} c_i \le x\}\, . \tag{9}$$

Then:

$$F_C(x,t) = \sum_{n=0}^{\infty} P\{N(t) = n\} F_n(x)\, .$$

Using the exponential distribution with the parameter ψ for the distribution of costs of failure causes and consequences clearance and using the Poisson distribution with the parameter λt for the distribution of the number of failures up to time *t* we get:

$$P\{N(t) = n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!}\, ,$$

$$f_0(x) = \delta(x),\quad F_0(x) = 1 \Leftrightarrow x > 0,\quad F_0(x) = 0 \Leftrightarrow x \le 0\, ,$$

$$f_1(\mathrm{x}) = \psi \mathrm{e}^{-\psi x},\quad F_1(x) = 1 - e^{-\psi x}\, ,$$

where f denotes the density function, $\delta(x)$ denotes the Dirac function and

$$f_n(x) = \frac{\psi^n x^{n-1} e^{-\psi x}}{(n-1)!},\quad \mathrm{F_n}(x) = \frac{\psi^n \int_0^x \varsigma^{n-1} e^{-\psi\varsigma}\, d\varsigma}{(n-1)!}$$

are the characteristics of the Gamma distribution.

If we use the notation $G_n(t) = e^{-\lambda t}\dfrac{(\lambda t)^n}{n!}$ and $\mathrm{G}_0(t) = e^{-\lambda t}$, we can write:

$$F_C(x,t) = \sum_{\mathrm{n=0}}^{\infty} \mathrm{G_n}(t) F_n(x)\, . \tag{10}$$

We can easily derive the following iterative method for individual parts of the sum (the definition domain of all mentioned functions is the positive half-axle):

$$G_0(t) = e^{-\lambda t},\ F_0(x) = 1,\ h_0(x) = e^{-\psi x}\, ,$$

$$h_{n+1}(x) = \frac{\psi x}{n+1} h_n(x),$$

$$F_{n+1}(x) = F_n(x) - h_n(x),$$

$$G_{n+1}(t) = \frac{\lambda\, t}{n+1} G_n(t). \qquad (11)$$

These equations (11) underline the effective numeric methods which enable us to compute the majority of required computations in acceptable time (of course using current information technologies).

4 Identification of parameters

Using the following assumptions:

$\tau_1, \tau_2, \tau_3, ..., \tau_{i-1}, \tau_i, \tau_{i+1}, ...$ are (iid) and have exponencial distribution with the parameter λ,

$c_1, c_2, c_3, ..., c_{i-1}, c_i, c_{i+1}, ...$ have the same probability distribution with the parameter ψ and

$\delta_1, \delta_2, \delta_3, ..., \delta_{i-1}, \delta_i, \delta_{i+1}, ...$ are (iid) and have exponencial distribution with the parameter ω,

we can apply the same procedure for identification of the required parameters. We can use the Maximum likelihood method with the estimation of the parameter $\overline{\lambda}$:

$$\overline{\lambda} = n \Big/ \sum_{i=1}^{n} \tau_i \,,$$

if n observations from the end of previous outage up to the beginning of the following one are available. Identification of the rest parameters will follow the same procedure, as it is obvious from the assumptions mentioned above.

5 Non-classical model with one time (one common clock)

In some practical situations, there will not be usable the above mentioned assumptions either about a negligence of an outage durations toward the others times or about the existence of double model clock. Standard models exploit times up to a failure. Then the time axis will look like as follows:

$$\begin{array}{cccccccc} \tau_1, & \delta_1+\tau_2, & \delta_2+\tau_3, & \dots, & +\tau_i, & \delta_i+\tau_{i+1}, & \delta_{i+1}+\tau_{i+2}, & \dots \\ \theta_1, & \theta_2, & \theta_3, & \dots & \theta_i, & \theta_{i+1}, & \theta_{i+2}, & \dots \end{array} \qquad (12)$$

Durations between failures will be formed as a sum of two independent variables with exponential distribution (with the exeption of the first one), therefore:

$\theta_1 = \tau_1$ with the distribution $F_1(t) = 1 - e^{-\lambda t}$

and the density $f_1(t) = \lambda\, e^{-\lambda t}$ (13)

$\theta_i = \delta_{i-1} + \tau_i$ with the distribution $F_i(t) = 1 - \dfrac{\lambda}{\lambda - \omega} e^{-\omega t} + \dfrac{\omega}{\lambda - \omega} e^{-\lambda t}$

and the density $f_i(t) = g(t) = \dfrac{\lambda\omega}{\lambda - \omega}(e^{-\omega t} - e^{-\lambda t})$. (14)

Denoting $S_n = \sum_{i=1}^{n} \Theta_i$ the time up to n^{th} failure, i.e. the time up to the beginning of i^{th} outage, and

$H_n(t)$ the distribution function of the random variable S_n, then the following holds true:

$$H_0(t) = 1 \Leftrightarrow t \geq 0 \quad \text{and} \quad H_0(t) = 0 \Leftrightarrow t < 0\,;\, H_1(t) = F_1(t)$$

$$\text{and further } H_n(t) = \int_0^t H_{n-1}(t-\upsilon)g(\upsilon)d\upsilon \quad n > 1 \tag{15}$$

$$\text{and} \quad P\{N(t) = n\} = H_n(t) - H_{n+1}(t)\,. \tag{16}$$

Using formulas (15) and (16) we get the iteration for

$$P\{N(t) = n\} = \int_0^t P\{N(t-\upsilon) = n-1\}g(\upsilon)d\upsilon\,. \tag{17}$$

This formula is a classical expression of a probability, that just *n* failure events will occure up to time *t*. It is the analytic notation, hence for the practical usage with distributions from the formulas (13) and (14), we will have to replace it by an effective numeric instrument. If we use for the time axis the quantization with the length of a computational step Δ for the time axis, the trapezium rool for the numeric integration and for the quantized time $t = k\Delta$, then we can write:

$$P\{N(k\Delta) = n\} = \Delta \sum_{j=1}^{k} P\{N((k-j)\Delta) = n-1\}g(j\Delta)\,. \tag{18}$$

It is quite simply and for the main fast executable algorithm. It is necessary to assign an initial condition, i. e. the probability that no failure will occure up to time *t*:

$$P\{N(t) = 0\} = H_0(t) - H_1(t) = e^{-\lambda t}\,. \tag{19}$$

Thereby we have prepared all the instruments for modelling of the processes *C(t)* and *D(t)* from the formulas (7) and (8) employing conditions, that we deal with only one time and with the standard real time clock:

$$F_C(x,t) = \sum_{n=0}^{\infty} P\{N(t) = n\}F_n(x)\,. \tag{20}$$

This formula has the same form for both the cumulative costs of failure causes and consequences clearance and the cumulative time of outage durations. Assigning to the one or the latter indicator is possible only by substituting the relevant distribution function $F_n(x)$ to the formula, or more exactly, in our case, by choosing the parameters ψ for cumulative costs and ω for cumulative outage durations.

6 Conclusion

We put forward the variations of the classical model of cumulated claims for modelling of processes of operation and outages of real working equipments. We complete the analytic models for various assumptions with computational instruments in the form of iterations (11) or quite simple discrete convolutions (18). Our models are based on the simplest class of recovery processes with the combination of reporting of costs (expressed first by own economic costs and the second by the power equipment outage duration).

7 Discussion and potential model variation

Our paper describes one possible version of solving the proposed problem. Of course one can also vary mentioned assumptions. The probability

$P\{N(t)=n\}=e^{-\lambda t}\frac{(\lambda t)^n}{n!}$ can be replaced by $P\{N(t)=n\}=(1-p(t))p(t)^n$ where $p(t)=\frac{\lambda\,\mathrm{t}}{1+\lambda\,\mathrm{t}}$, i.e. by the geometric distribution with the same mean value as the classic Poisson distribution. In such case we can achieve even closed formulas for expressions of type (20). The numerical method used for solving the equation (17) is not the only possible. One can achieve elegant results by using the Laplace or Fourier transformation. Of course the question of the classification of our model can be discussed. The name of our paper leads to the relation to insurance mathematics. We can not exclude relations of our model to the reliability theory (strong links) and also to the queuing theory (weaker links). According to questions we put in our paper, it is very close to insurance mathematics; according to problems solved in our paper, it is close to the reliability theory. As for the practical usage of our model we refer to the research report [9].

References

[1] Albrecher, H.: *Dependent Risks and Ruin Probabilities in Insurance.* Interim Report. IIASA, IR-98-072/September.
[2] Aven, T., Jensen, U.: *Stochastic Models in Reliability.* Springer 1998.
[3] Hátle, J., Likeš, J.: *Základy počtu pravděpodobnosti a matematické statistiky.* SNTL Praha 1974.
[4] Hurt, J.: *Teorie spolehlivosti.* Skripta MFF UK, Praha 1984.
[5] Mongomery, D.C.: *Introduction to Statistical Quality Control*. Wiley, New York 1991.
[6] Nahmias, S.: *Production and Operations Analysis.* IRWIN, Boston 1989.
[7] Rao, C. R.: *Lineární metody statistické indukce a jejich aplikace*. Praha, Academia, 1978.
[8] Rényi, A.: *Teorie pravděpodobnosti*. Praha, Academia, 1972.
[9] *Theoretic Model of Maintenance Optimization and Management of Transmission System Equipment in ČEPS Inc., Research Report, IV. phase for ČEPS Inc.* New Technologies Research Centre, University of West Bohemia, Pilsen, September 2002.

Current Address

Doc. Ing. František Vávra, CSc., vavra@kiv.zcu.cz, tel.: +420-377-491-157
Ing. Pavel Nový, Ph.D., novyp@kiv.zcu.cz
Ing. Arnoštka Netrvalová, netrvalo@kiv.zcu.cz
Ing. Michala Kotlíková, kotlikov@kiv.zcu.cz
Ing. Hana Mašková, maskova@kiv.zcu.cz

Department of Computer Science and Engineering, Faculty of Applied Sciences,University of West Bohemia ,Univerzitní 22, 306 14 Plzeň, Czech Republic

Tel.: +420-377376250, +420-377491212